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S. No. of Question Paper : 8308

Unique Paper Code : 235166

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Name of the Paper : MAPT-101 : Calculus and Matrices

Name of the Course : B.Sc. (Hons.) Computer Science/

B.Sc. Mathematical Sciences/Physical Sciences Part I

Semester : I

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any two questions from each Section.

Use of non-programmable scientific calculator is permitted.

### Section I

1. (a) Show that the set  $S = \{(1,3), (1,-1)\}$  forms a basis for  $\mathbb{R}^2$ .  
(b) Solve the system of equations :

$$x + y + z = 6$$

$$2x + 3y + 4z = 20$$

$$x + y = z$$

- (c) Examine if the set  $S = \{(a, b, 0) : a, b \in \mathbb{R}\}$  is a subspace of  $\mathbb{R}^3$ . If yes, justify and give geometrical interpretation of the subspace. 4,4,4

P.T.O.

2. (a) Is the transformation defined by  $T(x, y, z) = (x - y, y + z)$  linear? Justify.  
 (b) Find the characteristic equation, eigen values and eigen vector (corresponding to one of

them) for the matrix  $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$ .

- (c) Using elementary row operations, find the inverse of the matrix  $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ . 4,4,4

3. (a) Find the image of a triangle with vertices (1, 1), (3, 1) and (2, 3) under a dilation of factor 2.

- (b) Reduce the matrix  $\begin{bmatrix} 2 & 4 & 6 \\ -1 & 4 & -2 \\ -1 & 10 & -1 \end{bmatrix}$  to triangular form by elementary row operations and hence determine its rank.

- (c) Express the vector (3, 7) as a linear combination of the vectors (1, 3) and (0, 1) of  $\mathbb{R}^2$ . 4,4,4

### Section II

4. (a) Discuss the convergence of the following sequences :

(i)  $\langle (-1)^n \rangle$

(ii)  $\langle \frac{\cos^2 n}{3^n} \rangle$ .

- (b) Sketch the graph of the function  $y = 1 - \cos x$ ,  $x \in [0, 2\pi]$ . Mention the transformations used at every step.

- (c) A body at temperature  $72^{\circ}\text{F}$  is taken outdoors, where the temperature is  $20^{\circ}\text{F}$ . After 5 minutes, the temperature of the body is  $55^{\circ}\text{F}$ . How long will it take the body to reach a temperature of  $43^{\circ}\text{F}$  ? 6,6,6

5. (a) If  $y = \ln(x + \sqrt{1+x^2})$ , prove that  $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2y_n = 0$ .
- (b) Assuming the possibility of expansion, write the Maclaurin's series expansion up to at least five terms of the function  $f(x) = e^{2x}$ .
- (c) If  $v = \log(x^2 + y^2 + z^2)$ , prove that :

$$x \frac{\partial^2 v}{\partial y \partial z} = y \frac{\partial^2 v}{\partial z \partial x} = z \frac{\partial^2 v}{\partial x \partial y} . \quad 6,6,6$$

6. (a) Find the  $n$ th derivative of  $y = \frac{1}{(x^2 + 1)}$ .
- (b) Verify that the function  $u(x, t) = Px + Q$ , where P and Q are constants, is a solution to the heat equation.
- (c) Find the limit of the following sequences :

(i)  $\left\langle \frac{\log n}{n} \right\rangle$

(ii)  $\left\langle \left(1 + \frac{2}{n}\right)^n \right\rangle$ .

6,6,6

P.T.O.

## Section III

7. (a) If  $\cos \alpha + \cos \beta + \cos \gamma = \sin \alpha + \sin \beta + \sin \gamma$ , prove that :
- $$\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3 \cos(\alpha + \beta + \gamma).$$
- (b) Find the equation of a circle described on the line joining the points given by  $-1 - 3i$  and  $5 + 7i$  as diameter. 4,3½
8. (a) Use De Moivre's Theorem to solve the equation  $z^5 - 1 = 0$ .
- (b) Show that the points  $1 + 6i$ ,  $3 + 10i$ , and  $4 + 12i$  are collinear. 4,3½
9. (a) Form an equation of lowest degree with real coefficients having  $2 + 3i$  and  $3 + 5i$  as two of its roots.
- (b) Find all the values of  $(1+i)^{2/3}$ . 4,3½